

10.5: 4, 11-16, 20, 22, 32(a)

4. $\mathbf{r}(s, t) = s \sin 2t \mathbf{i} + s^2 \mathbf{j} + s \cos 2t \mathbf{k}$, so the corresponding parametric equations for the surface are $x = s \sin 2t$, $y = s^2$, $z = s \cos 2t$. For any point (x, y, z) on the surface, we have $x^2 + z^2 = s^2 \sin^2 2t + s^2 \cos^2 2t = s^2 = y$. Since no restrictions are placed on the parameters, the surface is $y = x^2 + z^2$, which we recognize as a circular paraboloid whose axis is the y -axis.
11. $\mathbf{r}(u, v) = \cos v \mathbf{i} + \sin v \mathbf{j} + u \mathbf{k}$. The parametric equations for the surface are $x = \cos v$, $y = \sin v$, $z = u$. Then $x^2 + y^2 = \cos^2 v + \sin^2 v = 1$ and $z = u$ with no restriction on u , so we have a circular cylinder, graph IV. The grid curves with u constant are the horizontal circles we see in the plane $z = u$. If v is constant, both x and y are constant with z free to vary, so the corresponding grid curves are the lines on the cylinder parallel to the z -axis.
12. $\mathbf{r}(u, v) = u \cos v \mathbf{i} + u \sin v \mathbf{j} + u \mathbf{k}$. The parametric equations for the surface are $x = u \cos v$, $y = u \sin v$, $z = u$. Then $x^2 + y^2 = u^2 \cos^2 v + u^2 \sin^2 v = u^2 = z^2$, which represents the equation of a cone with axis the z -axis, graph V. The grid curves with u constant are the horizontal circles we see, corresponding to the equations $x^2 + y^2 = u^2$ in the plane $z = u$. If v is constant, x, y, z are each scalar multiples of u , corresponding to the straight line grid curves through the origin.
13. $\mathbf{r}(u, v) = u \cos v \mathbf{i} + u \sin v \mathbf{j} + v \mathbf{k}$. The parametric equations for the surface are $x = u \cos v$, $y = u \sin v$, $z = v$. We look at the grid curves first; if we fix v , then x and y parametrize a straight line in the plane $z = v$ which intersects the z -axis. If u is held constant, the projection onto the xy -plane is circular; with $z = v$, each grid curve is a helix. The surface is a spiraling ramp, graph I.
14. $x = u^3$, $y = u \sin v$, $z = u \cos v$. Then $y^2 + z^2 = u^2 \sin^2 v + u^2 \cos^2 v = u^2$, so if u is held constant, each grid curve is a circle of radius u in the plane $x = u^3$. The graph then must be graph III. If v is held constant, so $v = v_0$, we have $y = u \sin v_0$ and $z = u \cos v_0$. Then $y = (\tan v_0)z$, so the grid curves we see running lengthwise along the surface in the planes $y = kz$ correspond to keeping v constant.
15. $x = (u - \sin u) \cos v$, $y = (1 - \cos u) \sin v$, $z = u$. If u is held constant, x and y give an equation of an ellipse in the plane $z = u$, thus the grid curves are horizontally oriented ellipses. Note that when $u = 0$, the "ellipse" is the single point $(0, 0, 0)$, and when $u = \pi$, we have $y = 0$ while x ranges from $-\pi$ to π , a line segment parallel to the x -axis in the plane $z = \pi$. This is the upper "seam" we see in graph II. When v is held constant, $z = u$ is free to vary, so the corresponding grid curves are the curves we see running up and down along the surface.
16. $x = (1 - u)(3 + \cos v) \cos 4\pi u$, $y = (1 - u)(3 + \cos v) \sin 4\pi u$, $z = 3u + (1 - u) \sin v$. These equations correspond to graph VI: when $u = 0$, then $x = 3 + \cos v$, $y = 0$, and $z = \sin v$, which are equations of a circle with radius 1 in the xz -plane centered at $(3, 0, 0)$. When $u = \frac{1}{2}$, then $x = \frac{3}{2} + \frac{1}{2} \cos v$, $y = 0$, and $z = \frac{3}{2} + \frac{1}{2} \sin v$, which are equations of a circle with radius $\frac{1}{2}$ in the xz -plane centered at $(\frac{3}{2}, 0, \frac{3}{2})$. When $u = 1$, then $x = y = 0$ and $z = 3$, giving the topmost point shown in the graph. This suggests that the grid curves with u constant are the vertically oriented circles visible on the surface. The spiraling grid curves correspond to keeping v constant.
20. $x = 4 - y^2 - 2z^2$, $y = y$, $z = z$ where $y^2 + 2z^2 \leq 4$ since $x \geq 0$. Then the associated vector equation is $\mathbf{r}(y, z) = (4 - y^2 - 2z^2) \mathbf{i} + y \mathbf{j} + z \mathbf{k}$.

22. In spherical coordinates, parametric equations are $x = 4 \sin \phi \cos \theta$, $y = 4 \sin \phi \sin \theta$, $z = 4 \cos \phi$. The intersection of the sphere with the plane $z = 2$ corresponds to $z = 4 \cos \phi = 2 \Rightarrow \cos \phi = \frac{1}{2} \Rightarrow \phi = \frac{\pi}{3}$. By symmetry, the intersection of the sphere with the plane $z = -2$ corresponds to $\phi = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$. Thus the surface is described by $0 \leq \theta \leq 2\pi$, $\frac{\pi}{3} \leq \phi \leq \frac{2\pi}{3}$.

32. (a) Here $z = a \sin \alpha$, $y = |AB|$, and $x = |OA|$. But

$$|OB| = |OC| + |CB| = b + a \cos \alpha \text{ and } \sin \theta = \frac{|AB|}{|OB|} \text{ so that}$$

$$y = |OB| \sin \theta = (b + a \cos \alpha) \sin \theta. \text{ Similarly } \cos \theta = \frac{|OA|}{|OB|} \text{ so}$$

$$x = (b + a \cos \alpha) \cos \theta. \text{ Hence a parametric representation for the}$$

$$\text{torus is } x = (b + a \cos \alpha) \cos \theta, y = (b + a \cos \alpha) \sin \theta,$$

$$z = a \sin \alpha, \text{ where } 0 \leq \alpha \leq 2\pi, 0 \leq \theta \leq 2\pi.$$

